Precision measurement of $\pi\pi$ scattering lengths at NA48/2

Andrea Bizzeti
University of Modena and Reggio Emilia
and I.N.F.N. Sezione di Firenze, Italy

on behalf of the NA48/2 collaboration:
Cambridge, CERN, Chicago, Dubna, Edinburgh, Ferrara, Firenze, Mainz, Northwestern, Perugia, Pisa, Saclay, Siegen, Torino, Vienna

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Introduction

The NA48/2 experiment at CERN SPS: setup and data

$K^{\pm} \to \pi^{+}\pi^{-}e^{\pm}\nu$ (K_{e4}) decays: Form Factors and $\pi\pi$ scattering lengths

NA48/2 K_{e4} and $K_{3\pi}$ results: comparison

Conclusions

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Why to measure $\pi\pi$ scattering lengths

- a_0 and a_2 are the S-wave $\pi\pi$ scattering lengths in isospin states I=0 and I=2
- They are related to fundamental parameters of Chiral Perturbation Theory (ChPT)
- Theoretical calculations based on ChPT provide accurate predictions for their values
- A precise measurement of a_0 and a_2 allows to test the theory and provides important constraints for the ChPT Lagrangian parameters

How to measure $\pi\pi$ scattering lengths

3 kinds of measurements have been developed:

Pionium atoms: DIRAC (CERN SPS) $(\pi\pi)$ lifetime

PLB 619 (2005) 50-60

$K_{3\pi}$ modes (cusp): $\mathrm{BR}(K^{\pm} \to \pi^{\pm}\pi^{0}\pi^{0}) = (1.757 \pm 0.024) \cdot 10^{-2}$

$\longrightarrow$ NA48/2 (CERN SPS): $60 \cdot 10^{6}$

EPJC 64 (2009) 589-608

$\mathrm{BR}(K_L^0 \to \pi^{0}\pi^{0}\pi^{0}) = (19.56 \pm 0.14) \cdot 10^{-2}$

KTeV (FNAL Tevatron). $68 \cdot 10^{6}$

NA48 (CERN SPS): $100 \cdot 10^{6}$

K_{e4} decays: $\mathrm{BR}(K^{\pm} \to \pi^{+}\pi^{-}e^{\pm}\nu\ ) = (4.09 \pm 0.09) \cdot 10^{-5}$

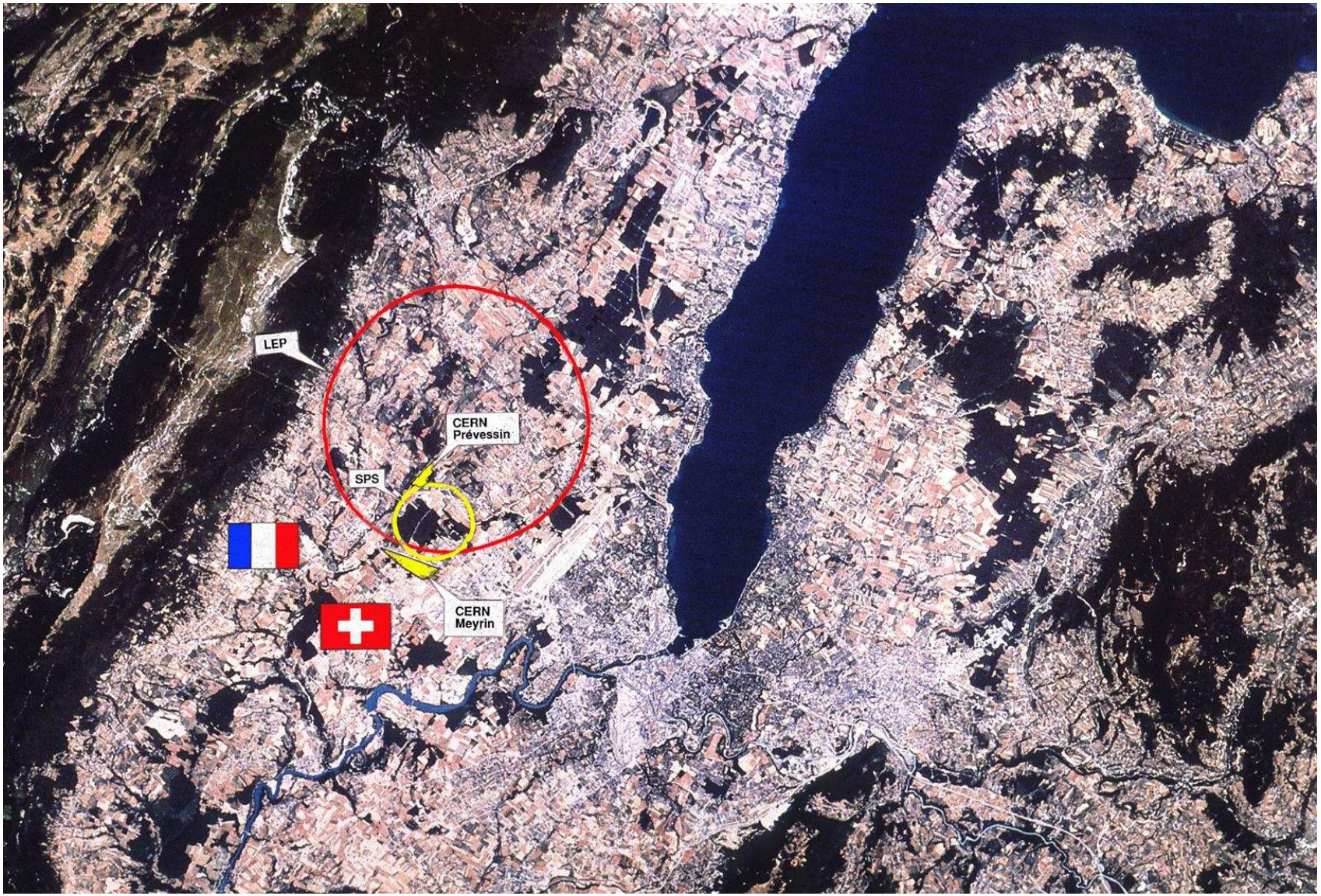
Very clean environment, but limited statistics:

S118 (CERN PS): $0.03 \cdot 10^{6}$

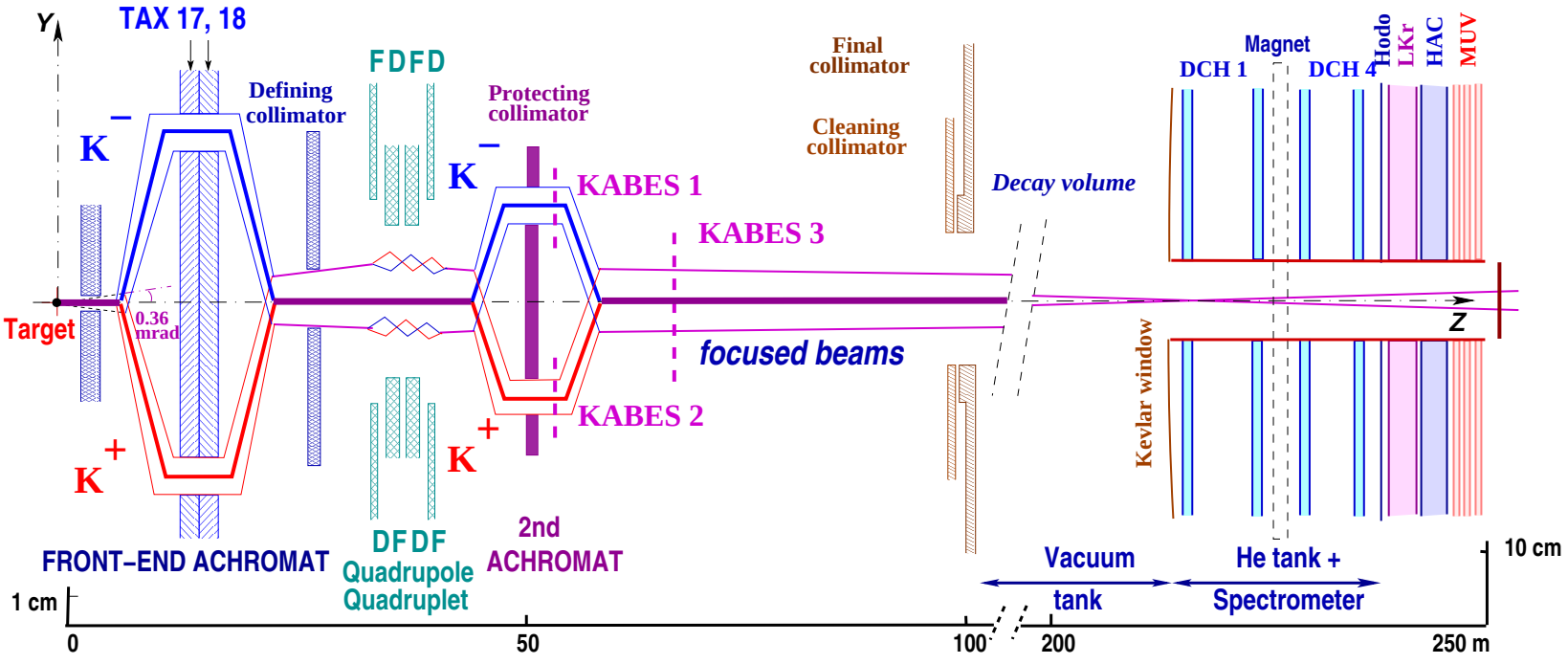
E685 BNL experiment $0.4 \cdot 10^{6}$

$\longrightarrow$ NA48/2 (CERN SPS): $1.1 \cdot 10^{6}$

The NA48/2 experiment at CERN SPS

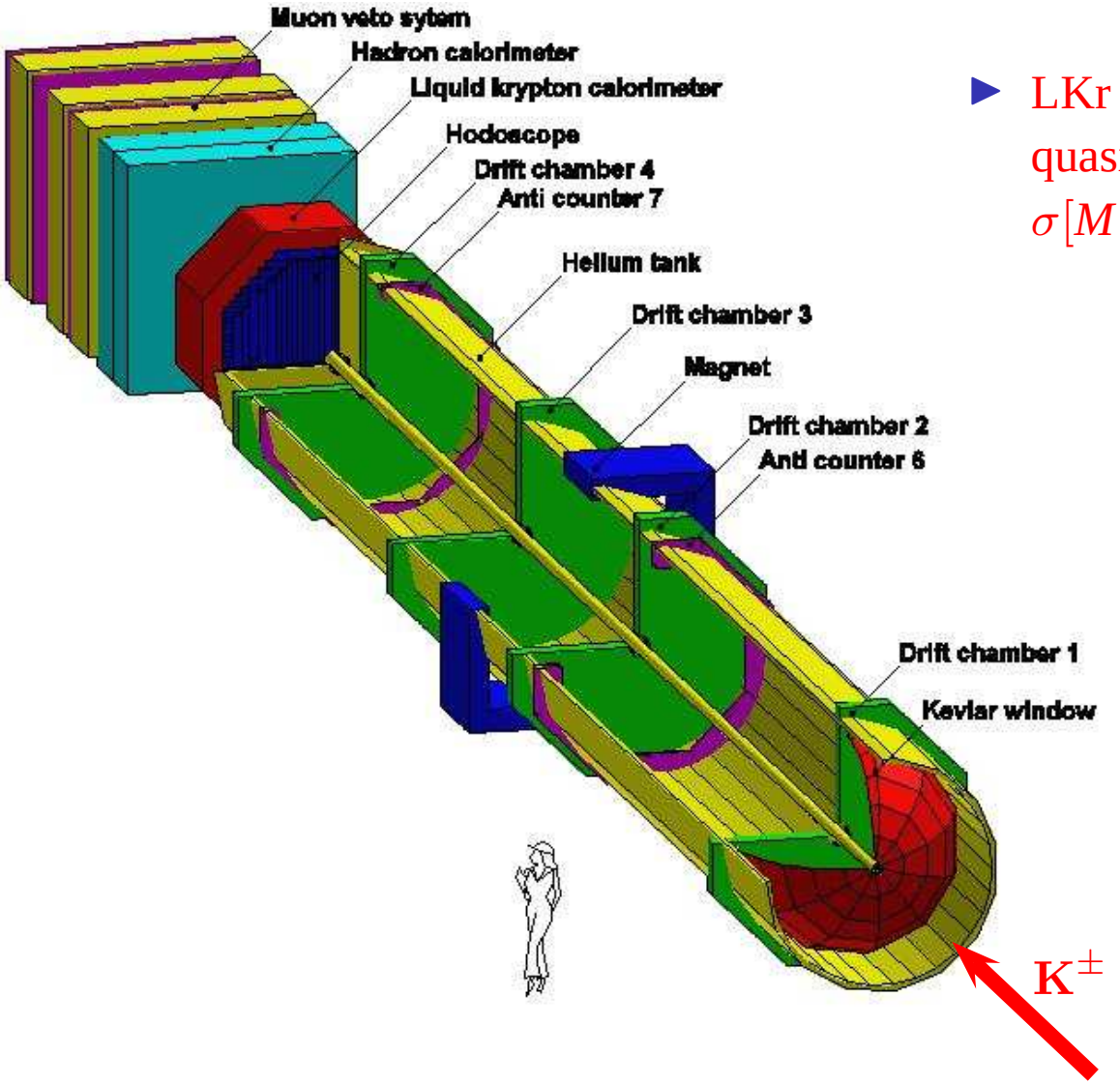


The NA48/2 beams



- Simultaneous [$P = (60 \pm 3)$ GeV/c] K^+ and K^- beams
  $\Rightarrow$ large charge symmetrization of experimental conditions
- Beams coincide within $\sim$ 1 mm along the 114 m decay volume.
- Flux ratio $K^+/K^- \sim 1.8$.

The NA48/2 detectors



- LKr electromagnetic calorimeter: quasi-homogeneous, high granularity $\sigma[M(\pi^{\pm}\pi^{0}\pi^{0})] = 1.4$ MeV/c^2
  - Magnetic spectrometer: 4 DCH + dipole magnet $\sigma[M(3\pi^{\pm})] = 1.7$ MeV/c^2
  - $\Rightarrow$ e/π discrimination (E/p)
  - Scintillator hodoscope for charged fast trigger: $\sigma(t) = 150$ ps
  - hadron calorimeter
  - muon counters
  - photon vetoes

The NA48/2 Data sample

Main goal of NA48/2:
search for CP violation in $K^{\pm} \to 3\pi$ Dalitz plots

Two years of data taking: 2003 run ($\sim$ 50 days)
+ 2004 run ($\sim$ 60 days)

Total statistics:

- $K^{\pm} \to \pi^{+}\pi^{-}\pi^{\pm}$: $\sim 4 \cdot 10^{9}$
- $K^{\pm} \to \pi^{0}\pi^{0}\pi^{\pm}$: $\sim 1 \cdot 10^{8}$
- $K^{\pm} \to \pi^{+}\pi^{-}e^{\pm}\nu$: $\sim 1.13 \cdot 10^{6}$

$\pi\pi$ scattering lengths measurement from $K_{3\pi}$ decays ("cusp")
already published $\to$ EPJ C 64 (2009) 589-608

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Ke4 decays: kinematics and formalism

Five kinematic variables (Cabibbo-Maksymowicz 1965):

$s_\pi = M^2_{\pi\pi}$, $s_e = M^2_{e\nu}$,
$\cos\theta_\pi$, $\cos\theta_e$, ϕ

Partial Wave expansion of the amplitude into s and p waves
+ Watson theorem for δ_l^I
$\Rightarrow \delta_0^0 = \delta_s$ and $\delta_1^1 = \delta_p$

2 Axial Form Factors (F and G):

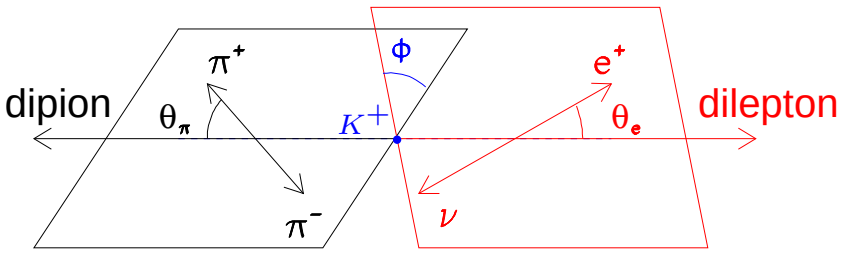
$F = F_s\, e^{i\delta_s} + F_p\, e^{i\delta_p} \cos\theta_\pi$

$G = G_p\, e^{i\delta_p}$

1 Vector Form Factor (H):

$H = H_p\, e^{i\delta_p}$

K^+(at rest) $\longrightarrow$ $\pi^+\pi^-$ $e^+\nu$



Fit the distribution in the five-dimensional space of the Ca.Ma. variables with 4 form factors and only one phase shift, assuming identical phases for the p-wave form factors

The fit parameters are:
F_p F_p G_p H_p and $\delta = \delta_s - \delta_p$
(F_p, F_p, G_p, H_p, δ are real)

Ke4 decay: Event selection and background rejection

Signal ($\pi^+\pi^-e^\pm\nu$) topology:

- 3 charged tracks and a “good” vertex
- 2 opposite sign pions, 1 electron [$E_{LKr}/p \simeq 1$]
- some missing energy and p_T (ν)
- good reconstructed P_K (missing ν hypothesis)

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Background main sources:

- $K^+ \to \pi^+\pi^-\pi^+$ ($\pi^+ \to e^+\nu$ or π^+ mis-ID)
- $K^+ \to \pi^+\pi^0$ ($\pi^0 \to e^+e^-\gamma$ and e^- mis-ID)

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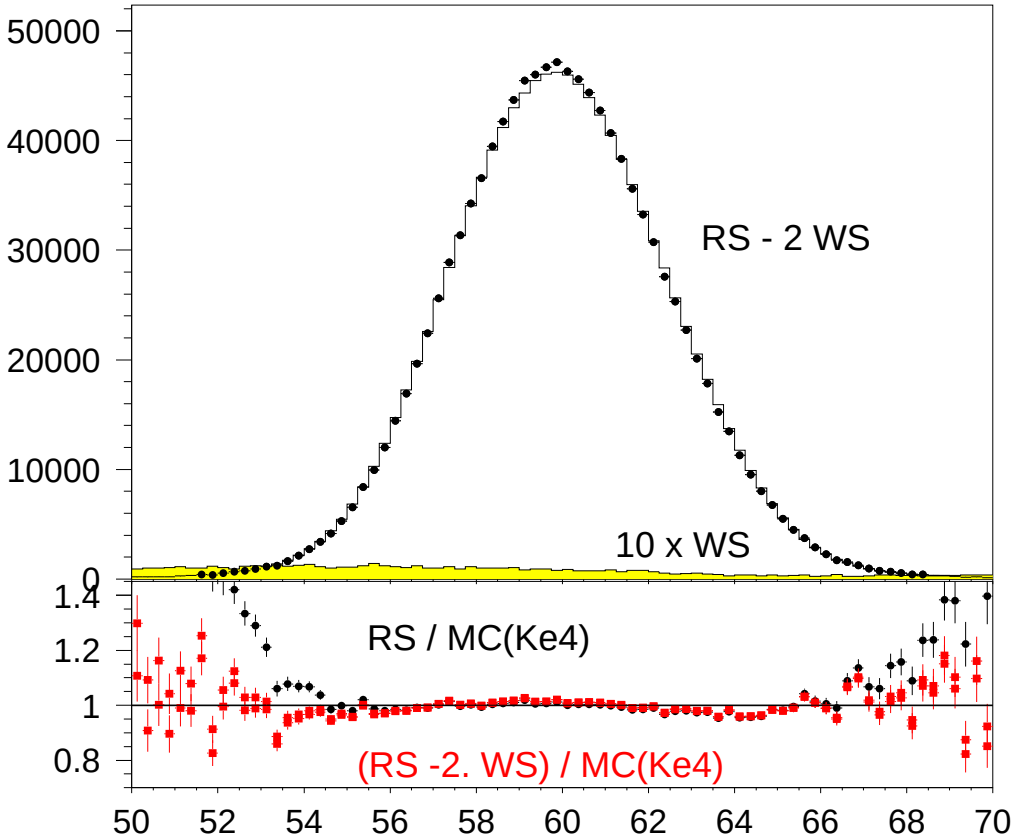
Control sample from data (assuming $\Delta S = \Delta Q$):

- $K^\pm \to \pi^\pm\pi^\pm e^\mp\nu$ ("Wrong-Sign" events)
  2 same sign pions + 1 electron
- Ratio RS/WS ("Right-Sign"/"Wrong-Sign") events =
  - 2/1 if coming from $K_{3\pi}$ (dominant)
  - 1/1 if coming from $K_{2\pi}$

Ke4 decay: background rejection

Total background level $\sim 2 \times 0.3\%$

- estimated from WS events in Data
- checked with Monte Carlo simulation of background processes



Reconstructed Kaon momentum (GeV/c)

Data/MC ratio

black: no BKG subtraction

red: subtraction = $2 \times$ WS

Ke4 decay: fitting procedure

Total (2003+2004) 1.13 million K_{e4} decays

Using iso-populated boxes in the 5-D space of the Ca.Ma variables ($M_{\pi\pi}$, $M_{e\nu}$, $\cos\theta_\pi$, $\cos\theta_e$ and ϕ) we define a grid of

$$\boxed{10 \times 5 \times 5 \times 5 \times 12 = 15000 \text{ variable size boxes}}$$

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In each $M_{\pi\pi}$ "slice" (containing 1500 boxes):

- a set of 4 fit parameters (F_p, G_p, H_p, δ) is extracted
- the normalization F_s^2 is obtained by the ratio Data / MC

	Data sample (events)	Monte Carlo sample (events)
K_{e4}^+	726 400 (48 / box)	17.4 million (1160 / box)
K_{e4}^-	494 400 (27 / box)	9.7 million (650 / box)

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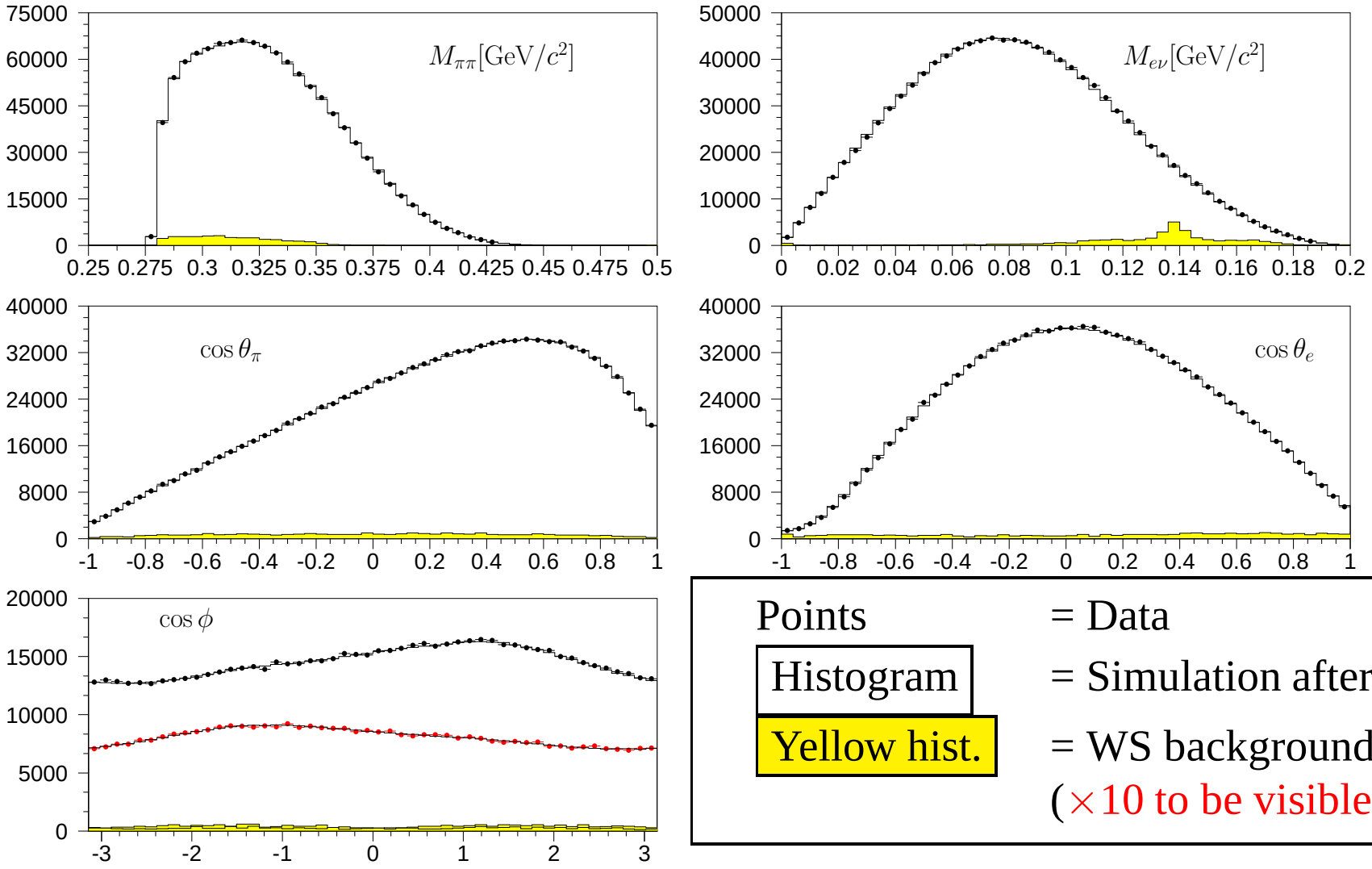
K^+ and K^- samples fitted separately in 10 independent $M_{\pi\pi}$ slices, then combined in each slice according to their statistical error.

No assumption is made on the variation of the phase δ (and FF) from one $M_{\pi\pi}$ slice to the next (i.e. "model independent" analysis)

Ke4 decay: Data / MC comparison (after the fit)



Points	= Data
Histogram	= Simulation after fit
Yellow hist.	= WS background ($\times$10 to be visible)

Ke4 Form Factors: fit results

A series expansion in $q^2 = [M^2_{\pi\pi}/(4m^2_\pi) - 1]$ and $M^2_{e\nu}/(4m^2_\pi)$ is used to describe the FF variations in the isospin symmetry limit :

$$F_s^2 = f_s^2[1 + f_s'/f_s \; q^2 + f_s''/f_s \; q^4 + f_e'/f_s \; M_{e\nu}/(4m_\pi^2)]^2$$

$$G_p/f_s = g_p/f_s + g_p'/f_s \; q^2$$

$$F_p = f_p \qquad H_p = h_p$$

Systematics:

- mostly from background + acceptance control
- comparable or smaller than statistical error

Total statistics (2003+2004)

	value	stat	syst
f_s'/f_s	0.152	$\pm$0.007	$\pm$0.005
f_s''/f_s	-0.073	$\pm$0.007	$\pm$0.006
f_e'/f_s	0.068	$\pm$0.006	$\pm$0.007
f_p/f_s	-0.048	$\pm$0.003	$\pm$0.004
g_p/f_s	0.868	$\pm$0.010	$\pm$0.010
g_p'/f_s	0.089	$\pm$0.017	$\pm$0.013
h_p/f_s	-0.398	$\pm$0.015	$\pm$0.008

Ke4 decays: phase shifts and $\pi\pi$ scattering lengths

The relation between the fitted $\delta = \delta_s - \delta_p$ phase shift and the $\pi\pi$ scattering length a_0 and a_2 can be predicted from data above 0.8 GeV using Roy equations (unitarity, analicity and crossing symmetries). Numerical solutions have been developed, valid only in the Isospin symmetry limit.

$\Rightarrow$ Need to take into account **isospin breaking**.

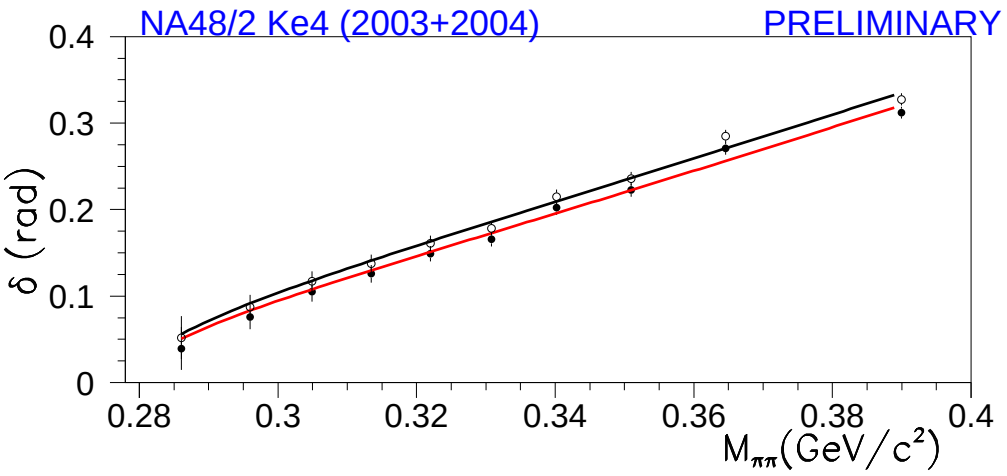
Electromagnetic effects

- Gamow-Sommerfeld factor:
  "classical" Coulomb attraction between 2 charged pions
- PHOTOS generator:
  real photon(s) emitted and tracked in the simulation

Mass effects ($m_u \neq m_d$, $m_{\pi^+} \neq m_{\pi^-}$)

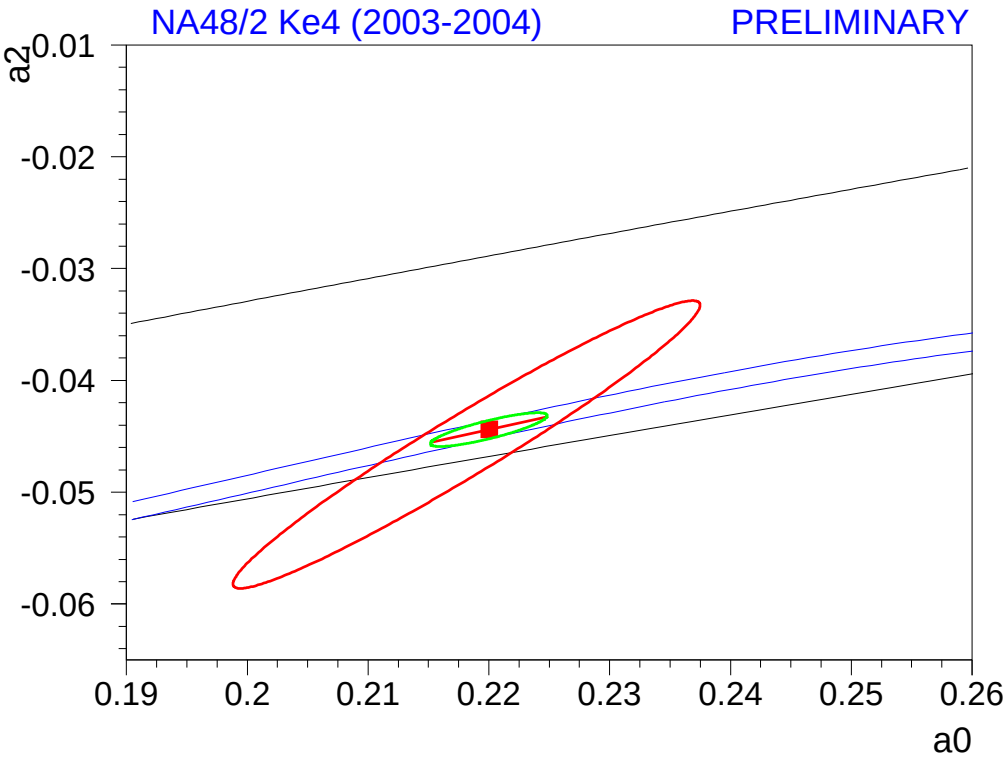
- recently computed as a correction to the measurements
- larger than current experimental precision!
  (CGR, EPJ C59(2009) 777)

Ke4 decays: phase shifts and scattering lengths



Black: <u>without</u> isospin breaking corrections

Red: <u>with</u> isospin breaking corrections



Red ellipse: 2–parameter fit

Blue band: ChPT constraint

CGL, NPB 603 (2001) 125

CGL, PRL 86 (2001) 5008

Green ellipse: 1-parameter fit (using ChPT constraint)

$\longleftarrow \quad \times(1/m_{\pi^+})$

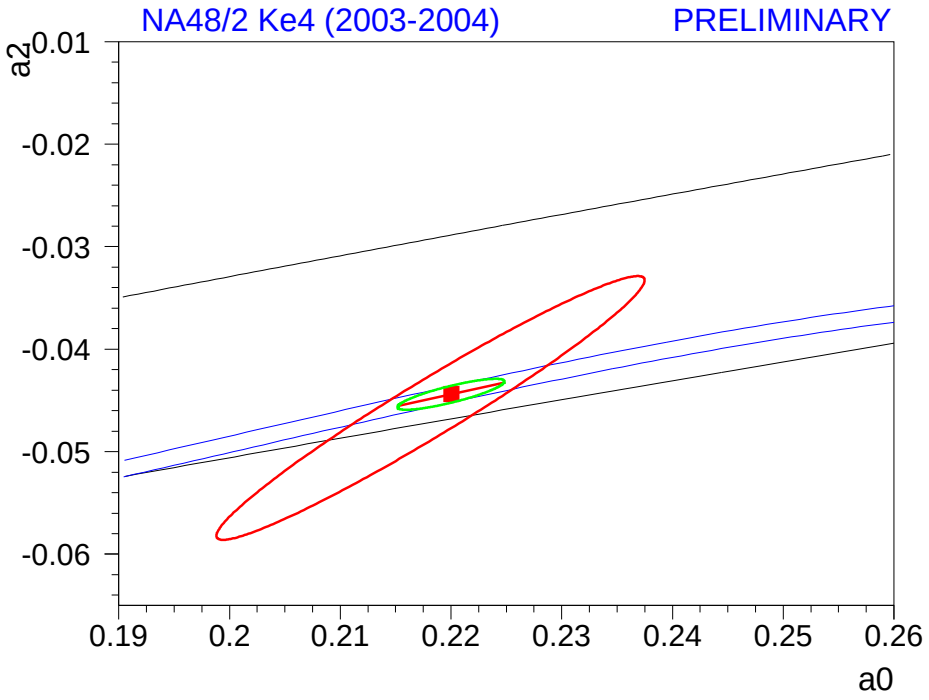
Ke4 decays: comparison with theoretical predictions

NA48/2 experimental results (in $1/m_{\pi^+}$ units)

2-par. Fit: $a_0 = 0.2220 \pm 0.0128_{\text{stat}} \pm 0.0050_{\text{syst}} \pm 0.0037_{\text{th}}$

(correl. ρ=0.967) $a_2 = -0.0432 \pm 0.0086_{\text{stat}} \pm 0.0034_{\text{syst}} \pm 0.0028_{\text{th}}$

1-par. Fit (ChPT): $a_0 = 0.2206 \pm 0.0049_{\text{stat}} \pm 0.0018_{\text{syst}} \pm 0.0064_{\text{th}}$



Theory prediction

Assuming more inputs from ChPT and low energy constants (CGL NPB603(2001), PRL86(2001))

$a_0 = 0.220 \pm 0.005$

$a_2 = -0.0444 \pm 0.0008$

K_{e4} and $K_{3\pi}$(cusp) results comparison

Two independent measurements

- K_{e4}: $1.13 \cdot 10^6$ events
- Cusp: $60 \cdot 10^6$ $K_{3\pi}$ $(K^{\pm} \to \pi^0\pi^0\pi^{\pm})$ events

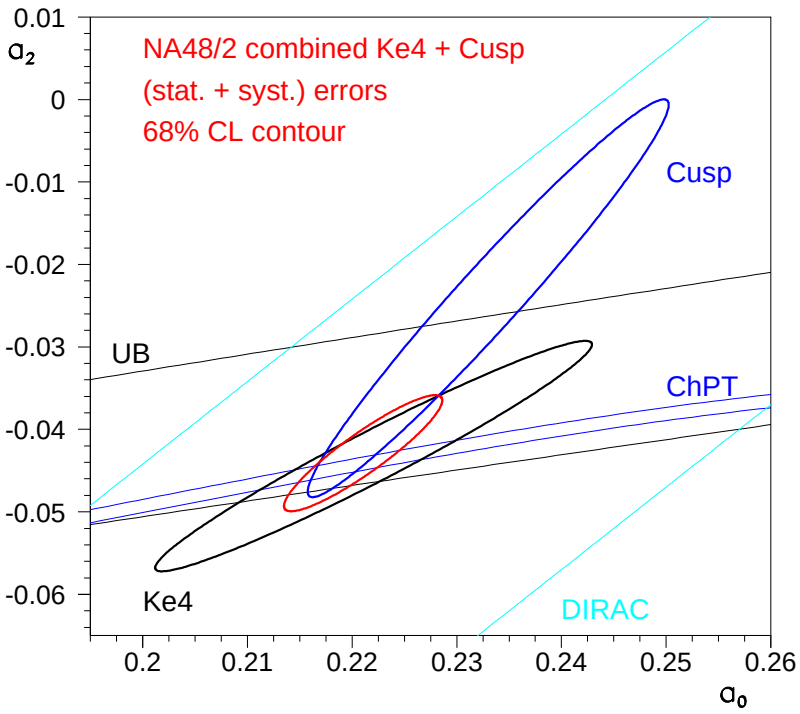
Different systematics

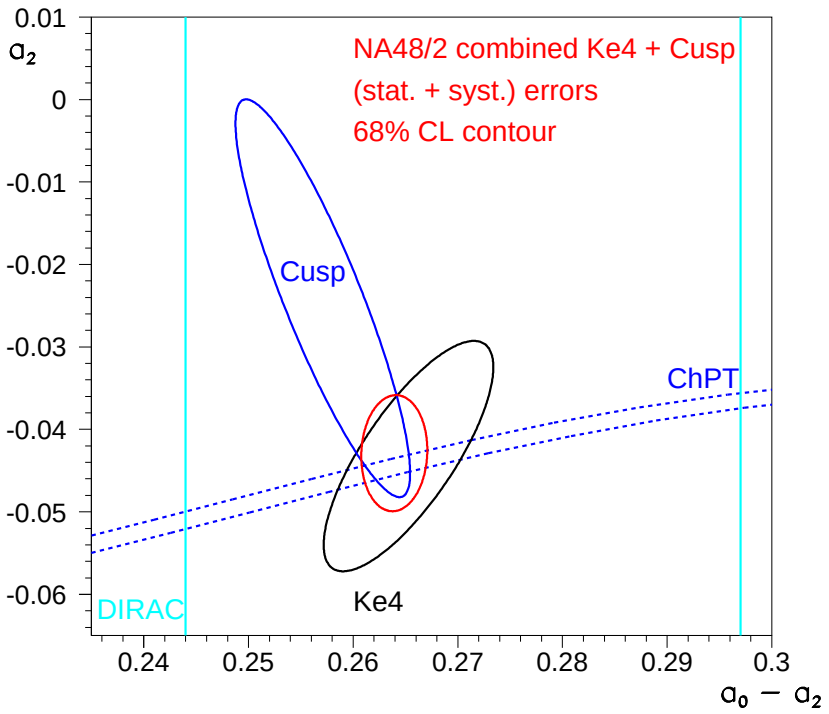
- K_{e4}: electron mis-ID and Background
- Cusp: Calorimeter and Trigger

Different theoretical inputs

- K_{e4}: Roy equations and Isospin breaking corrections
- Cusp: Final state rescattering and ChPT expansion

Combined NA48/2 results from K3pi(cusp) and Ke4





free (a_0, a_2)	stat	syst	theo
$a_0 =$	0.2210 ± 0.0047	± 0.0015	± 0.0049
$a_2 =$	-0.0429 ± 0.0044	± 0.0016	± 0.0030

Correlation 0.910

free $(a_0 - a_2, a_2)$	stat	syst	theo
$a_0 - a_2 =$	0.2639 ± 0.0020	± 0.0004	± 0.0021
$a_2 =$	-0.0429 ± 0.0044	± 0.0016	± 0.0030

Correlation 0.277

Including ChPT constraint: $a_2 = -0.0444 \pm 0.0007 \pm 0.0005\ (\pm 0.0012)$ stat/syst/(theo)

$a_0 = 0.2195 \pm 0.0027 \pm 0.0021\ (\pm 0.0048)$ or $a_0 - a_2 = 0.2640 \pm 0.0020 \pm 0.0017\ (\pm 0.0035)$

Total exp. errors: $\Delta a_0 = \pm 0.0034$, $\Delta a_2 = \pm 0.0009$, $\Delta(a_0 - a_2) = \pm 0.0026$

Conclusions

- The kaon decays give the possibility to study the low energy hadronic interaction with good precision
- Thanks to large statistics and high data quality, NA48/2 can check ChPT predictions with very high accuracy
- $\pi\pi$ scattering lengths from K_{e4} and $K_{3\pi}$ are fully consistent
- The achieved experimental precision on a_0 is now competitive with the theoretical precision (± 0.005)
- The two precise and independent measurements of $\pi\pi$ scattering lengths provide a very strong test of the theory